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# Nonpolynomial Normal Modes of the Renormalization Group in the Presence of a Constant Vector Potential Background

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**Abstract**

We examine the renormalization group flow in the vicinity of the free-field fixed point for effective field theories in the presence of a constant, nondynamical vector potential background. The interaction with this vector potential represents the simplest possible form of Lorentz violation. We search for any normal modes of the flow involving nonpolynomial interactions. For scalar fields, the inclusion of the vector potential modifies the known modes only through a change in the field strength renormalization. For fermionic theories, where an infinite number of particle species are required in order for nonpolynomial interactions to be possible, we find no evidence for any analogous relevant modes. These results are consistent with the idea that the vector potential interaction, which may be eliminated from the action by a gauge transformation, should have no physical effects.

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In quantum theory, the electromagnetic field is incorporated through the use of the vector potential $A^{\mu}$. While it is commonly held that only the electric and magnetic fields are physically relevant, the vector potential itself can contribute to the physics in curious ways [1]. If the vector potential is in a pure gauge configuration, $A^{\mu}(x) = \partial^{\mu}\Omega(x)$, then the electromagnetic field tensor $F^{\mu\nu}$ vanishes; however, the nonzero $A^{\mu}$ still appears in the action, through the coupling to charged matter. We shall examine the effects of this apparently trivial coupling on the renormalization group (RG) flows for effective field theories.

The presence of a nonvanishing $A^{\mu}$ in the vacuum state is actually the simplest possible form of Lorentz violation, and the possibility of this and other forms of Lorentz violation has recently received a significant amount of attention. There has been a great deal of perturbative work examining the general structure of the relevant quantum field theories [2, 3, 4] and the one-loop quantum corrections [5, 6, 7, 8, 9, 10, 11]. Small violations of Lorentz symmetry may arise in a low-energy effective field theory as remnants of larger violations appearing in a fundamental theory at higher energies. So it is natural to study Lorentz violation in the context of an effective theory with a cutoff. We shall examine the renormalization group flow for theories of this sort, in the vicinity of the Gaussian fixed point. Our results will be nonperturbative in the Lorentz-violating parameter. By studying the RG trajectories, we shall be considering some further effects of quantum corrections.

In particular, we shall be concerned with any nonpolynomial interactions that might arise in an effective field theory. Halpern and Huang [12, 13], using the Wilsonian formulation of the RG [14, 15, 16], have shown that in scalar field theories, there exist relevant nonpolynomial interactions; and Periwal [17], using exact RG methods [18], has derived a regularization-independent differential equation governing the RG flow for these interactions.

This paper is organized as follows: We first present a simplified and improved derivation of the differential equation governing the linearized RG flow in the scalar field case. Our treatment makes clear the role of tadpole loops in determining the RG flow. We then generalize this method to include the possibility of a constant, nonvanishing vector potential, interacting with either bosons or fermions. In the bosonic case, we find that the presence of the vector potential causes the form of the nonpolynomial interactions to undergo an apparent change. However, we also show that this change may be cancelled out by a redefinition of the coupling constant. In the fermionic generalization, which requires the presence of an infinite number of fermion species, we find no nonpolynomial modes generated by the presence of the potential. We conclude with a discussion of the significance of these results, as well as several subtle points related to our calculations.

**Potential-free theory—** We begin by re-examining the Halpern's and Huang's original result. We shall enumerate all the Feynman diagrams that contribute to the RG flow at lowest order and then derive the partial differential equation governing the coupling

constants' evolution. We find that the dominant loop contributions come entirely from tadpole diagrams. Although we use Feynman diagrams, our results are nonperturbative, because we shall be summing an infinite number of diagrams, and the integration momenta in the tadpole loops are correlated at all orders.

We first consider a theory with a single real scalar field; the generalization to multiple scalar fields is simple. We shall investigate the quantum corrections to this theory using bare perturbation theory. The Euclidean action for the unrenormalized scalar theory is

$$S_b = \int d^d x \left[ \frac{1}{2} (\partial \phi)^2 + V_b \left( \phi^2 \right) \right], \tag{1}$$

where the interaction Lagrange density $V_b$ is representable as a power series in $\phi^2$. From this bare action, we may determine the renormalized effective action, which arises from loop corrections. We shall use a momentum cutoff $\Lambda$ to regulate the loop integrals that arise in the calculation of the effective action.

To determine the full effective action, we need to find the coefficient of each effective vertex of the theory, and to determine the exact coefficient of an effective $n$-particle vertex, we need to calculate the full $n$-particle correlation function from the bare theory. However, we shall only be interested in a restricted class of interactions—those which involve no derivatives (i.e. are independent of the external momenta) and which are at most linear in the bare couplings. We may therefore make an approximation and omit some Feynman diagrams in our determination of the $n$-particle correlation function. When we neglect the momentum-dependent interactions and the interactions that are nonlinear in $V_b$, we find that the contributions to the effective $n$-particle amplitude have a distinct form. Written as a Feynman diagram, each of these contributions contains a bare $(n+2k)$-point vertex and $k$ tadpole loops. The lowest-order diagrams contributing to the four-particle amplitude are shown in Fig. 1.

We can sum up all diagrams of this sort without difficulty [19]. Each loop contributes a factor of $\frac{1}{2} D_F(0)$, where $D_F(x-y)$ is the Feynman propagator for a massless scalar particle; $D_F$ differs from minus the inverse Laplacian only through its large-momentum regulation. The factor of 2 is the symmetry factor for the loop. There is also an additional symmetry factor of $k!$ corresponding to interchanging of the $k$ loops.

We want to express the effective coupling constants in a dimensionless fashion, so we write the effective interaction action as

$$S_{b,int} = \int d^d x \, \Lambda^d U_b \left[ \Lambda^{-(d-2)/2} \phi \right]. \tag{2}$$

The function $U_b \left[ \Lambda^{-(d-2)/2} \phi \right]$ also depends upon $\Lambda$ as a parameter; this parametric $\Lambda$-dependence represents the dimensionally anomalous behavior of the effective potential. When we total up the combinatorial factors, we find that, with the approximations mentioned above, the effective potential is given by

$$\Lambda^d U_b \left[ \Lambda^{-(d-2)/2} \phi \right] = \sum_{k=0}^{\infty} \frac{1}{k!} \left[ \frac{1}{2} D_F(0) \frac{\partial^2}{\partial \phi^2} \right]^k V_b \left( \phi^2 \right). \tag{3}$$

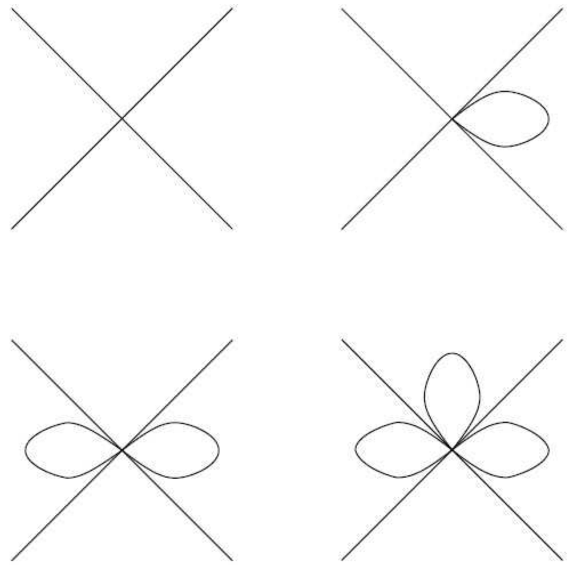

Figure 1: Graphs contributing to the four-particle amplitude.

In dimensions $d > 2$, the regulated value of $D_F(0)$ has the form $C_b\Lambda^{d-2}$, for some constant $C_b$. The explicit value of $C_b$ is given in [12]. (When $d = 2$, the dependence on $\Lambda$ is logarithmic, but analogous results continue to hold.) If we insert this expression into (3), we have

$$\Lambda^d U_b\left[\Lambda^{-(d-2)/2}\phi\right] = \exp\left[\frac{1}{2}C_b\Lambda^{d-2}\frac{\partial^2}{\partial\phi^2}\right]V_b\left(\phi^2\right). \tag{4}$$

If we apply the operator $\Lambda\frac{\partial}{\partial\Lambda}$ to both sides of (4) and then use (4) to rearrange the right-hand side, we find

$$\Lambda\frac{\partial U_b}{\partial\Lambda} + dU_b - \frac{d-2}{2}\Lambda^{-(d-2)/2}\phi U_b'\left[\Lambda^{-(d-2)/2}\phi\right] = \frac{d-2}{2}C_b U_b''\left[\Lambda^{-(d-2)/2}\phi\right], \tag{5}$$

which is essentially just Periwal's differential equation. We can see that the right-hand side, which determines the quantum corrections to the RG flow, has come entirely from tadpole contributions. The solutions of the differential equation (5) with power-law dependences on $\Lambda$—that is, with $\Lambda\frac{\partial U_b}{\partial\Lambda} = -\lambda U_b$— are eigenmodes of the RG flow near the Gaussian fixed point. When $\lambda > 0$, these potentials describe theories that are asymptotically free, with much stronger scale dependences than are seen in superficially renormalizable theories. All these relevant directions correspond to nonpolynomial interactions.

The complex perturbative structure of these theories is outlined in [17]. Of particular importance is the fact that the anomalous dimension $\lambda$ describes only how the potential scales with respect to the cutoff $\Lambda$; it does not describe the scaling of any correlation functions with respect to their external momenta.

**Bosonic theory with $A^\mu$**— We now generalize the theory to include a constant, nondynamical electromagnetic potential $A^\mu(x) = a^\mu$, interacting with a complex scalar field $\Phi$. The vector $a$ represents a preferred direction in spacetime and is the source of the breaking of (Euclidean) Lorentz invariance. The action becomes

$$S_b = \int d^d x \left\{ \left[ \left( \partial_\mu - i a_\mu \right) \Phi^* \right] \left[ \left( \partial^\mu + i a^\mu \right) \Phi \right] + V_b\left( 2|\Phi|^2 \right) \right\}. \tag{6}$$

The modified Lagrange density involves two new terms. These are the interaction term $ia^\mu \left[ \left( \partial_\mu \Phi^* \right) \Phi - \Phi^* \left( \partial_\mu \Phi \right) \right]$ and a shift in the mass term $a^2 |\Phi|^2$.

The preferred vector $a$ can be absorbed into the definition of the field, according to

$$\Phi \rightarrow e^{-ia\cdot x} \Phi. \tag{7}$$

If the theory is defined perturbatively in $a$, then this gauge transformation removes $a$ from the theory entirely [3]. (However, in the presence of multiple particle species, interacting with multiple vector potentials, the differences between the values of $a$ corresponding to different species can be observable [20, 21, 22].) Since $a$ can be removed in this fashion, the operator $ia^\mu \left[ \left( \partial_\mu \Phi^* \right) \Phi - \Phi^* \left( \partial_\mu \Phi \right) \right]$ cannot receive quantum corrections at any order in perturbation theory and therefore cannot have a nonzero anomalous dimension. (This is explicitly verified to one-loop order in [10] for the analogous fermionic theory that we shall examine later in this paper.) The RG flow of $a$ is thus purely classical. We may therefore write $a$ as $a = \hat{a}\theta\Lambda$, where $\theta$ is a scale-independent number, and $\hat{a}$ is a unit vector in the direction of $a$.

Although $a$ can be eliminated by a field redefinition, there are still some subtleties associated with it. If the magnitude of $a$ is too large, then the quantization of the theory encounters difficulties [4], unless the field redefinition is performed before the quantization procedure. Since we are not eliminating $a$ before we quantize the theory, we must insist that the magnitude of $a$ be small compared to some relevant mass scale, which we shall take to be the cutoff $\Lambda$. This implies that $\theta \ll 1$.

The field redefinition (7) does not leave the renormalization prescription invariant (a consequence of the familiar fact that a momentum cutoff regulator is not gauge invariant), so it is conceivable that the Halpern-Huang modes may be modified by the presence of this interaction. The key quantity that determines the structure of these modes is $D_F(0)$. In four dimensions, and in the presence of $a$, this becomes

$$D_F(0) = \int_{|p|<\Lambda} \frac{d^4 p}{(2\pi)^4} \frac{1}{(p-a)^2}. \tag{8}$$

This modification of $D_F(0)$ is the only effect of $a$ that we shall consider.

The modified $D_F(0)$ may be evaluated by separating the integration variable $p$ into portions $p_\parallel$ and $p_\perp$, which are parallel and normal to $a$, respectively. Then the propagator

at zero separation becomes

$$D_F(0) = \frac{1}{(2\pi)^4}\int_{-\Lambda}^{\Lambda} dp_\parallel \int_0^{\sqrt{\Lambda^2-p_\parallel^2}} dp_\perp \frac{4\pi p_\perp^2}{p_\perp^2+(p_\parallel-a)^2} \tag{9}$$

$$= \frac{\Lambda^2}{4\pi^3}\left[\frac{\pi}{2}-\int_{-1}^{1} dy\,(y-\theta)\left(\tan^{-1}\frac{\sqrt{1-y^2}}{y-\theta}\right)\right], \tag{10}$$

where $y = p_\parallel/\Lambda$. For $\theta = 0$, $D_F(0) = \Lambda^2/16\pi^2$; for nonvanishing $a$, the integral cannot be evaluated exactly.

Although we cannot obtain a closed-form expression for $D_F(0)$ when $\theta$ is nonvanishing, we can define a new constant

$$C_b(\theta) = \frac{D_F(0)}{\Lambda^2} = \frac{1}{16\pi^2} + \frac{3}{32\pi^2}\theta^2 + \mathcal{O}(\theta^4). \tag{11}$$

Then we may repeat our earlier arguments and obtain the linearized RG equations in the presence of $a$. The only essential difference (aside from those differences following from the presence of two real scalar fields) involves the replacement $C_b \to C_b(\theta)$. So the normal modes of the RG flow in the vicinity of the free field fixed point are exactly as found by Halpern and Huang, except that they involve the modified $C_b(\theta)$.

The nonpolynomial normal modes are therefore described by

$$V_b\left(2|\Phi|^2\right) = \Lambda^d U_b\left[\sqrt{2}\Lambda^{-(d-2)/2}\Phi\right], \tag{12}$$

where $U_b$ is the nondimensionalized potential given by

$$U_b(y) = gM\left[\frac{\lambda-d}{d-2};1;\frac{y^2}{2C_b(\theta)}\right]; \tag{13}$$

here, $g$ is a coupling constant, and $M(\alpha;\beta;z)$ is the confluent hypergeometric (Kummer) function [23]

$$M(\alpha;\beta;z) = 1 + \frac{\alpha}{\beta}\frac{z}{1!} + \frac{\alpha(\alpha+1)}{\beta(\beta+1)}\frac{z^2}{2!} + \cdots. \tag{14}$$

We see that, in the presence of the background potential $a$, the functional forms of the normal modes in fact differ from the forms they take when $a$ vanishes. However, this difference is not physically meaningful. It is clear that the difference appears only in the field strength renormalization. By changing the normalization of the fields, we may absorb the ratio $C_b(0)/C_b(\theta)$ into the coupling constant $g$. Alternatively, we might change the renormalization prescription slightly, cutting off the $p$-integration in $D_F(0)$ at $|p| = \Lambda\sqrt{C_b(0)/C_b(\theta)}$, rather than simply at $|p| = \Lambda$. Such a change in the cutoff will not affect the theory, except through a change in the coupling. This is all in keeping with the results found in [19], where the field strength renormalization in the presence of nonpolynomial potentials is studied in detail.

That the change in the form of the potential can be eliminated by rescaling the couplings is unsurprising, in light of the perturbative triviality of the interaction we have introduced. However, it is reassuring to see that the expected triviality persists in these nonperturbative results. Moreover, we can further illuminate the common structure of this triviality by looking at the behavior of the coupling constant $g$ in another particular context. As we have seen, the field strength renormalization that restores the potential to the Halpern-Huang form involves a finite multiplicative shift in the coupling. This is similar to what we see when we use the gauge transformation (7) to remove the operator $ia^{\mu}\left[\left(\partial_{\mu}\Phi^{*}\right)\Phi-\Phi^{*}\left(\partial_{\mu}\Phi\right)\right]$ from an otherwise free Lagrange density. As well as eliminating the interaction of $a$ with the charged current, (7) also introduces a shift in the mass parameter of the theory. So in this case also, the elimination of the nonstandard interaction changes the coupling constant of the theory by a finite factor.

**Fermionic theory—** We shall now turn our attention to the analogous fermionic theories. In order to generalize the bosonic case to include a nondynamical vector potential, we needed to work with a complex scalar field or, equivalently, two real scalar fields. However, our fermionic generalization necessarily requires that we introduce an *infinite* number of fields. Otherwise, the anticommutivity of the fermion fields will prevent the existence of nonpolynomial potentials. So we see that a generalization to fermion systems will necessarily be rather complicated.

The presence of a nonvanishing $a$ (or some other preferred vector) is also necessary for any generalization to fermions. The curious RG flows in the Halpern-Huang interaction directions arise from the factors of $D_F(0)=C_b\Lambda^{d-2}\neq 0$ that appear in loop diagrams. In a Lorentz-invariant theory, the corresponding quantity for Dirac fermions,

$$S_F(0)=i\int\frac{d^dp}{(2\pi)^d}\frac{\not{p}}{p^2}, \tag{15}$$

vanishes. However, in a theory containing one or more preferred vectors, $S_F(0)$ need not be zero (although its trace still must vanish). Then there may again be nontrivial relevant directions near the Gaussian fixed point.

So we shall consider a fermionic theory with unrenormalized Euclidean action

$$S_f=\int d^dx\left[\psi^{\dagger}\left(\not{\partial}+i\not{a}\right)\psi+V_f\left(i\psi^{\dagger}\psi\right)\right]. \tag{16}$$

$\psi$ and $\psi^{\dagger}$ are Grassmann-valued fields, and the $D$-dimensional Euclidean Dirac matrices are anti-hermitian and obey $\{\gamma_\alpha,\gamma_\beta\}=-2\delta_{\alpha\beta}I_D$, where $I_D$ is the identity matrix in spinor space. (We must eventually take the limit $D\rightarrow\infty$, to represent the infinite number of particle species.) Complex conjugation does not reverse the order of the Grassmann numbers—$(\alpha\beta)^*=\alpha^*\beta^*$. The function $V_f$ is real valued and must be expandable as a power series. With these conventions (which follow [24]), the action is real: $S_f^*=S_f$. We shall again regulate our theory with a momentum cutoff $\Lambda$ for any loop integrals. All the

arguments regarding the perturbative triviality of $a$ and its effects on the quantization of the theory apply exactly as in the bosonic case.

We may now directly generalize the methods we used to arrive at the differential equation governing the RG flow for bosons. We again evaluate the relevant propagator at zero separation. In the presence of $a$, we have, for $d > 1$,

$$S_F(0) = i \int_{|p|<\Lambda} \frac{d^d p}{(2\pi)^d} \frac{\not{p} - \not{a}}{(p-a)^2} = i\, \hat{\not{a}}\, C_f(\theta) \Lambda^{d-1}. \tag{17}$$

In four dimensions, the constant $C_f(\theta)$ is given by

$$\begin{aligned} C_f(\theta) &= \frac{1}{4\pi^3}\left[-\frac{\pi}{2}\theta - \int_{-1}^{1} dy\,(y-\theta)^2 \left(\tan^{-1}\frac{\sqrt{1-y^2}}{y-\theta}\right)\right] & (18)\\ &= -\frac{1}{32\pi^2}\theta + \mathcal{O}(\theta^3). & (19)\end{aligned}$$

Again, this modification of the zero-separation propagator is the only effect of $a$ that we shall consider.

The fact that $S_F(0)$ is not zero might allow for the existence of novel, asymptotically free theories analogous to those found in the bosonic case. Following this analogy, we look for an effective action with an interaction term of the form

$$S_{f,int} = \int d^d x\, \Lambda^d U_f \left[\Lambda^{-(d-1)} i\psi^\dagger\psi\right]. \tag{20}$$

Again, by assuming that $S_{f,int}$ has this form, we are neglecting momentum-dependent interactions. Moreover, we are also neglecting any interactions that have a nontrivial matrix structure in spinor space, such as $(\psi^\dagger\gamma\psi)^2$. Finally, we shall again neglect terms that are nonlinear in the bare couplings, since these do not affect the structure near the free-field fixed point.

We must now enumerate the contributing loop diagrams. Because the fermions have a nontrivial spinor structure, this is more complicated than in the scalar field case. However, we may utilize that fact that $D \to \infty$ to simplify our calculation. A nonvanishing fermion loop is a contraction of $\left(\psi^\dagger\psi\right)^{2m}$, such that the contractions form a single closed cycle. (We shall refer to this as a loop of size $2m$.) Each such loop contributes a factor of $D$, arising from a trace over the spinor space. As $D \to \infty$, only diagrams with loops of this form need be considered; all other topologies give contributions small in comparison. (However, those other topologies are critically important for fixed, finite $D$. In such a situation, the terms we have neglected ensure that there are no contributions from terms in $V_f$ with more than $D$ fermion operators.) There are $(2m-1)!$ ways to perform the contraction in constructing a loop of size $2m$; this is equivalent to a symmetry factor of $2m$, corresponding to the $2m$ cyclic permutations of the $\psi^\dagger\psi$ pairs that leave the contraction invariant. The value of a loop of size $2m$ is therefore $-\frac{1}{2m}\mathrm{tr}\left[S_F(0)\right]^{2m}$.

When we assemble the diagrams of interest, the loops of size $2m$ exponentiate as in the bosonic case, independently the loops of other sizes. We find that the effective interaction is therefore

$$\Lambda^d U_f\left[\Lambda^{-(d-1)}i\psi^\dagger\psi\right]=\exp\left\{-D\sum_{m=0}^{\infty}(-1)^m\frac{1}{2m}\left[C_f(\theta)\right]^{2m}\Lambda^{2m(d-1)}\frac{\partial^{2m}}{\partial\left(\psi^\dagger\psi\right)^{2m}}\right\}V_f\left(i\psi^\dagger\psi\right). \tag{21}$$

This leads to a differential equations analogous to (5); in terms of $x=\Lambda^{d-1}i\psi^\dagger\psi$, it is

$$\Lambda\frac{\partial U_f}{\partial\Lambda}+dU_f-(d-1)xU_f'(x)=-D(d-1)\left\{\sum_{m=0}^{\infty}\left[C_f(\theta)\right]^{2m}\frac{\partial^2}{\partial x^2}\right\}U_f(x). \tag{22}$$

If we take $U_f$ to be an eigenfunction of the RG flow, with $\Lambda\frac{\partial U_f}{\partial\Lambda}=-\lambda U_f$, and act on both sides of the differential equation with $\left[1-C_f(\theta)^2\frac{\partial^2}{\partial x^2}\right]$, then we have

$$\left\{\left[1-C_f(\theta)^2\frac{\partial^2}{\partial x^2}\right]\left[(\lambda-d)+(d-1)x\frac{\partial}{\partial x}\right]-(d-1)D\right\}U_f(x)=0. \tag{23}$$

This third-order differential equation has two solutions that are representable as power series about $x=0$. They can be expressed as generalized hypergeometric functions [25]

$$f_1^\lambda(x) = {}_1F_2\left[\frac{1}{2}\left(\frac{\lambda-d}{d-1}-D\right);\frac{1}{2}\left(\frac{\lambda-d}{d-1}+1\right),\frac{1}{2};\frac{x^2}{4C_f(\theta)^2}\right] \tag{24}$$

$$f_2^\lambda(x) = x\left\{{}_1F_2\left[\frac{1}{2}\left(\frac{\lambda-d}{d-1}-D+1\right);\frac{1}{2}\left(\frac{\lambda-d}{d-1}+2\right),\frac{3}{2};\frac{x^2}{4C_f(\theta)^2}\right]\right\}. \tag{25}$$

The function ${}_1F_2(\alpha;\beta,\gamma;z)$ (not to be confused with the more frequently occurring ${}_2F_1$) is defined to be

$${}_1F_2(\alpha;\beta,\gamma;z)=\sum_{i=0}^{\infty}\frac{\alpha(\alpha+1)\cdots(\alpha+i-1)}{\beta(\beta+1)\cdots(\beta+i-1)\cdot\gamma(\gamma+1)\cdots(\gamma+i-1)}\frac{z^i}{i!}. \tag{26}$$

The functions $f_1^\lambda(x)$ and $f_2^\lambda(x)$ do not have well-defined limits as $D\to\infty$ for any finite $\lambda$. Therefore, there can be no nonpolynomial interactions with power-law coupling constant flows (either relevant or irrelevant) generated in this manner. This is again in keeping with the triviality of $a$.

**Discussion—** There are still some subtle points to be addressed regarding our discussion of the fermionic case. We have neglected a large number of diagrams that could generate other interesting effects, and we must keep in mind that our differential equation (22) is valid only in the limit of infinite $D$. In fact, as $D\to\infty$, the equation itself does not have a well-defined limit. However, the solutions may have meaningful limits,

if $\lambda \to \infty$ also; this would appear to correspond to an infinitely rapid coupling constant flow. Specifically, if $\frac{\lambda-d}{d-1} = cD$ as $D$ approaches infinity, then $f_1^\lambda(x)$ and $f_2^\lambda(x)$ approach

$$f_1^\lambda(x) \quad \to \quad \cosh\left(\sqrt{\frac{c-1}{c}}\frac{x}{C_f(\theta)}\right) \tag{27}$$

$$f_2^\lambda(x) \quad \to \quad C_f(\theta)\sqrt{\frac{c}{c-1}}\sinh\left(\sqrt{\frac{c-1}{c}}\frac{x}{C_f(\theta)}\right). \tag{28}$$

Since (27) and (28) apparently correspond to interactions with infinitely rapid coupling constant flow, we might be inclined to conclude that no interaction $U_f(x) = g\exp(\alpha x)$ may be generated by radiative corrections, for the following reason: If such an interaction existed at some scale, then a finite shift in the cutoff $\Lambda$ could generate a new interaction that was infinitely strong, and hence unphysical. One might then further conclude that we had found a nontrivial effect caused by the presence of $a$. However, both conclusions are erroneous. The second conclusion is incorrect, because we have no *a priori* reason to believe that the exponential interactions in question are allowed even in the absence of a nonvanishing $a$; there might be other effects that prohibit their generation, unrelated to those discussed here. Moreover, even the first conclusion, that the potentials are forbidden, is incorrect. We have only considered the linearized coupling constant flows in the vicinity of the free-field fixed point. The RG flow for the exponential potential $U_f(x) = g\exp(\alpha x)$ necessarily carries the theory into the region of strong coupling, far from the fixed point, and so our treatment does not apply. We therefore cannot conclude anything about whether or not these interactions are allowed, and the question of whether the presence of $a$ may generate interactions of this particular form remains open.

Thus far, we have worked entirely in Euclidean space. If we transform these theories into Minkowski spacetime, we can draw some additional conclusions about the physics. An analytic continuation of $C_f(\theta)^2$ to Minkowski spacetime gives a result proportional to $\mathrm{sgn}(a^2)$, so, for lightlike $a$, there is clearly no possibility for any nonpolynomial interaction to be generated by the means we have discussed. This provides a partial answer to the question raised at the end of the previous paragraph.

Our entire discussion has been based on the use of a regulator that is not gauge invariant. This is not problematic in this instance, since the gauge field $A^\mu$ is constant and nondynamical; but it is questionable whether our methods would still be applicable in the presence of a nontrivial, quantized electromagnetic field. If we were required to use a gauge-invariant regulator, then $a$ would certainly not generate a change in the RG flow for any theory. However, since we have identified no nontrivial changes to either the bosonic or fermionic theories, our results are entirely consistent with this form of gauge invariance.

In this paper, we have provided some further confirmation of the triviality of $a$—the Lorentz-violating parameter corresponding to the vacuum expectation value of the vector

potential $A^\mu$. We have performed nonperturbative RG calculations in the presence of $a$, finding results entirely consistent with the triviality of this term. In the bosonic case, the only changes to the nonpolynomial Halpern-Huang interactions may be eliminated by a rescaling of the couplings. For the analogous fermionic system, there are no nonpolynomial modes in the vicinity of the free-field fixed point that possess power-law coupling constant flows. These results are consistent with the idea that the gauge parameter $a$ should have no measurable effects on any theory.

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